The intent of the book seems to be to collect in one place some mathematical results directly or indirectly relevant to finite element methodology. It can serve as a compact reference to these results, but it would be hard for students of mathematics to use this as a course text, without extensive supplements.

R.S.

38[65N99].—DEREK B. INGHAM & MARK A. KELMANSON, *Boundary Integral Equation Analyses of Singular, Potential, and Biharmonic Problems*, Lecture Notes in Engineering (C. A. Brebbia and S. A. Orszag, Editors), Springer-Verlag, Berlin, Heidelberg, New York, 1984, xiv + 173 pp., 24 cm. Price $12.50.

As the title of the book suggests, the purpose of this work is to extend the range of applicability of the boundary integral equation (BIE) method to include certain biharmonic problems and nonlinear potential problems, particularly problems which involve boundary singularities. The presence of these singularities greatly reduces the rate of convergence of standard BIE procedures. In their treatment of such problems, the authors modify the classical BIE method to take into account the analytic form of such a singularity, and demonstrate the efficacy of the modified method by comparing it with the classical BIE method on an appropriate test problem. In each problem discussed in this book, the approximate solution is piecewise constant, and, whenever possible, integrations are performed analytically, which, the authors claim, substantially reduces cpu time.

Chapter 1 is a brief introductory chapter. Chapter 2 is devoted to a discussion of the BIE solution of a biharmonic problem arising in fluid flow problems involving "stick-slip" boundary conditions, which give rise to a boundary singularity. In Chapter 3, methods similar to those developed in Chapter 2 are used to solve problems of flow near sharp corners, which involve corner singularities. Chapter 4 is concerned with certain nonlinear potential problems in which the differential equation can be linearized by applying the Kirchhoff transformation. The resulting problem not only has nonlinear boundary conditions but also boundary singularities. (It should be noted that Eq. (5) of this chapter is incorrect; the integral should be multiplied by φ^{-1}.) Viscous flow problems are also discussed in Chapters 5 and 6. In Chapter 5, problems with free surfaces, which are nonlinear, are considered, while Chapter 6 is devoted to a study of slow flow in bearings with arbitrary geometries. In Chapter 7, some conclusions are briefly stated. In Chapters 2 to 6, the results of numerical experiments are presented.

This book is an unusual publication. Not only is it a photo reproduction of the complete Ph.D. thesis of the second author, but five of its seven chapters, Chapters 2 to 6, have appeared in their entirety in five separate papers in refereed journals under the sole authorship of the second author. Since each of these chapters is self-contained, with its own abstract, introduction, conclusions, and references, there is substantial repetition, the elimination of which would considerably reduce the size of the book without any loss of information. Moreover, each of Chapters 2 to 6 is presented in the format of a preprint, with tables and figures gathered together after the references, and not inserted in the text where they are first mentioned. This, together with the lack of consistency in the notation, makes the book awkward to read.

There is no question that the material presented in this book is of much interest to researchers concerned with the development and application of BIE methods, particularly those interested in solving slow viscous flow problems. But it ought not to have been published in this form, when it has already appeared verbatim in the open literature.

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39[93B40, 93C20, 93C75].—K. L. TEO & Z. S. WU, *Computational Methods for Optimizing Distributed Systems*, Mathematics in Science and Engineering, Vol. 173, Academic Press, Orlando, Fla., 1984, xiii + 317 pp., $23\frac{1}{2}$cm. Price \$68.50.

With the rapid decrease of cost of computer power, the possibilities of using mathematical modelling in science and engineering have dramatically increased during the last decade. In particular, it is possible today to use computer-implemented numerical methods to solve complicated optimal control problems which may not be solved by classical analytical or ad hoc methods. This is particularly true for optimal control problems involving partial differential equations, and there is thus a great practical interest in having efficient numerical methods for such problems. There is a comparatively rich literature on theoretical-mathematical aspects of optimal control of partial differential equations, but much less so concerning numerical methods. The present book aims at partly filling this gap and is concerned with computational methods for optimal control of partial differential equations or distributed parameter systems. This problem area is very vast, including, in addition to analysis of the continuous problem, also discretization using finite element or finite difference methods and application of optimization methods for finite-dimensional problems, together with related convergence questions. The emphasis of the book in this wide spectrum is towards the continuous problem, particular attention being given to a mathematical convergence theory for certain gradient type methods for some optimal control problems involving (linear) parabolic equations. Discretized problems occur in the numerical examples, but are not treated theoretically. The material is based mainly on research of the authors and their associates.

A brief outline of the contents of the book is as follows: Chapters 1 and 2 contain background material from functional analysis and existence and regularity theory for linear parabolic partial differential equations. Chapter 3 is concerned with a class of optimal control problems involving linear parabolic equations with the controls occurring (nonlinearly) in lower-order derivative terms and in the forcing term. The controls are supposed to belong to a compact and convex subset of $\mathbb{R}^m$, and the cost functional is essentially a linear functional of the state at a given terminal time. This is a typical example of a stochastic optimal control problem with Markov terminal time. For this problem the authors consider a gradient type method involving, as is